AIRCRAFT LANDING AND ATTITUDE CONTROL USING DYNAMIC MATRIX CONTROL

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Abstract: This paper proposes a method for an efficient control of the aircraft landing and attitude through Dynamic Matrix Control. The idea of MPC structures used in aircraft control has been well established during the last few years, but some aspects require further investigation. With this in mind, the paper proposes structures for aircraft landing and aircraft attitude control by using single DMC controllers for landing and respectively one DMC controller for each of the attitude axis (pitch attitude hold, bank angle hold and heading hold). The model used for analysis of the aircraft landing structure is based on the last phase of landing. Also, the model used to illustrate the attitude control is that of a pitch attitude hold system of a N250-100 aircraft. Simulations are performed for a variety of control and prediction horizons, taking into account the possibility of adding a weighting factor for the control actions. Apart from separate studies on step reference variations, for some use cases, a generic reference trajectory is provided as a control purpose of the system. Results show a better performance of the proposed method in terms of control surface transition and protection of the actuators involved and a better time response in stabilizing the aircraft attitude. Overall, the aspects shown ensure an improved aircraft attitude control and landing stabilization.

Keywords: model predictive control, aircraft attitude control, DMC, flight control

1. Introduction

The Dynamic Matrix Control (DMC) algorithm is one of the first Model Predictive Control algorithms. Invented in 1980, by Cutler and Ramaker [1], the DMC algorithm has been used in a various range of applications because it is based on a linear step response model and an objective function given in quadratic form. This objective function is minimized over the given prediction horizon in order to compute the output of the optimal controller.

The original DMC formulation has been extended in numerous varieties in the last few years, [2]. For example, [3] proposes an adaptive control strategy based on multiple DMC models. Another application is presented in [4], illustrating the extension of the classical DMC formulation to nonlinear systems.

A few recent applications of DMC include the adaptive DMC with interpolated parameters [5], [6] and applications related to embedded real-time systems [7].

In [8], a detailed study about the prediction horizon influence in DMC is presented.

Dynamic Matrix Control structures has also been achieved using neural networks; for this, [9] presents a case study based on Elman neural networks.

Recent studies have also employed automatic code generation for DMC controllers as presented in [10].

Regarding applications in aircraft control, DMC has been applied in a limited number of cases. For example, an application regarding guidance navigation and control can be found in [11]. A more complex application implies self-tuning DMC of two-axis autopilot for small airplanes as can be seen in [12].

Since there are a limited number of applications in DMC for aircraft control, this paper proposes structures for landing stabilization and attitude control by applying simple single loop DMC controllers for the landing procedure and respectively for each of the attitude control subsystems (pitch attitude hold, bank angle hold and heading hold) which are part of the automated flight control system. The purpose of these applications is to obtain a more effective aircraft landing and attitude control and to ensure a more stabilized movement of the aircraft control surfaces.

2. Models and Methods

2.1. Dynamic Matrix Control

The objective of the DMC controller is to determine an appropriate set-point for a given criterion of the control actions. Monitoring the control actions limits the computational burden and prevents sudden variations of the output.

For a SISO system, with the step response given as, [13]:

\[ y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) \]  

(1)
The disturbance at time \( t \) over the prediction horizon is:
\[
x(t + j|t) = x(t|t) = e(t) \\
e(t) = y_m(t) - y(t|t)
\]
The predicted output is:
\[
y(t + j|t) = \sum_{i=1}^{j} g_i \Delta u(t + j - i) + x(t + j|t)
\]
\[\tag{2}\]
For constant disturbances, the predicted output value is:
\[
y(t + j|t) = \sum_{i=1}^{j} g_i \Delta u(t + N - i) + f(t + j)
\]
\[\tag{3}\]
where \( f(t + j) \) is the free response of the system which does not depend on the future control actions.

The prediction based on horizon \( N_p \) control actions is:
\[
y(t + N_p|t) = \sum_{i=N_p-N_c+1}^{N_p} g_i \Delta u(t + N - i) + f(t + N_p)
\]
\[\tag{4}\]
Equation (4) can be rewritten as, [13]:
\[
y = Gu + f
\]
\[\tag{6}\]
Equation (6) shows the relation between the future output and the control actions.

The dynamic matrix of the system \( G \) is formed from \( N_c \) (control horizon) columns of the step response ordered accordingly.

The predicted output together with the disturbances that affect it is given by:
\[
y_d = G_d u_d + f_d
\]
\[\tag{8}\]

The objective function that needs to be minimized is:
\[
J = \sum_{j=N_1}^{N_2} \delta(j)[y(t + j|t) - w(t + j)]^2 + \\
+ \sum_{j=1}^{N_c} \lambda(j)[\Delta u(t + j - 1)]^2
\]
\[\tag{9}\]
Where:
- \( N_p = [N_1, N_2] \) - prediction horizon
- \( \lambda \) - control action weighting matrix
- \( w \) - penalty terms matrix

The solution for this criterion can be obtained by deriving \( J \) and equaling it with 0, [14].
\[
u = (G^T + \lambda I)^{-1} G^T (w - f)
\]
\[\tag{10}\]

\[\lambda\]

The mathematical model which describes the aircraft during this flight phase demonstrates that it is unstable, this being shown also by the eigenvalues:

2.2. The Model of the Aircraft during its Last Phase of Landing

For the purpose of demonstrating DMC usage for aircraft landing stabilization the following model is used, [1], [15], representing an aircraft in the final phase of landing as can be seen in Fig. 1:

- The plane is located altitude of 30 m
- The aircraft horizontal speed is 77 m/s
- The roll and yaw movements are stabilized
- The initial descending speed (until the altitude of 30 m) is \( w_0 = -6m/s \)
- The descending speed must be maximum \( 0.6 m/s \) at the exact moment when the aircraft touches the runway, otherwise the landing gear might be affected.
- This landing phase lasts 20 sec
- The landing trajectory (in a vertical frame) is:

\[h(t) = \begin{cases} 30e^{-t/5} & 0 \leq t \leq 15 \\ 6 - t, 15 < t \leq 20 \end{cases}\]

\[\tag{12}\]

- The reference for the descending speed is:

\[w(t) = h(t) = \begin{cases} -6e^{-t/5} & 0 \leq t \leq 15 \\ -1.15 < t \leq 20 \end{cases}\]

\[\tag{13}\]

The mathematical model which describes the aircraft during this flight phase demonstrates that it is unstable, this being shown also by the eigenvalues:
\[ \lambda_1 = -0.5016 + j0.8670 \]
\[ \lambda_2 = -0.5016 - j0.8670 \]
\[ \lambda_3 = 0.0032 \]

In the control structure proposed in [15], a partial state feedback loop is constructed (for \( \omega_y \) and \( \Theta \)) and also a feedback loop for the output \( w \). This is shown in Fig. 2 below.

**Fig. 2.** Control system with a partial state feedback \((\omega_y\) and \(\Theta\)) and a feedback loop for the \(w\) output with \(S_c\) representing the compensated system as given in [15].

Given the state variables, a feedback loop is constructed, for which the feedback matrix \( K \) is obtained with the LQR algorithm, [15]:

\[
J = \int_{0}^{\infty} \left[ u^{T}(t)Q u(t) + u^{T}(t) V u(t) \right] dt \tag{15}
\]

\[
K = V^{-1} B^{T} P \tag{16}
\]

With the \( P \) matrix representing the solution of the Ricatti algebraic equation:

\[
PA + A^{T} P - PBV^{-1} B^{T} P + Q = 0 \tag{17}
\]

The compensated system has the following imposed eigenvalues:

\[
\lambda_1' = -8.1072 + j7.4184 \]
\[ \lambda_2' = -8.1072 - j7.4184 \]
\[ \lambda_3' = -0.3973 \tag{18} \]

The feedback matrix \( K \) has the following form:

\[
K = \begin{bmatrix} 6.5761 & 50.5295 & 0 \end{bmatrix} \tag{19}
\]

The state space representation for the compensated system is:

\[
A = \begin{bmatrix} -0.6 & -0.76 & 0.003 \\ 1 & 0 & 0 \\ 0 & 102.4 & -0.4 \end{bmatrix} \tag{20}
\]

\[
B = \begin{bmatrix} 2.374 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad D = 0
\]

Given the state space representation, an equivalent model can be calculated for the compensated system:

\[
A_c = A - BK \tag{21}
\]

This equivalent model is to be used as a basis for the calculations regarding the proposed control structures.

### 2.3. The Autopilot Model Used in Attitude Control

In the following, a theoretical pitch attitude control system is described. The model is used to show the advantages of the proposed control structure. The purpose is to design independent model predictive controllers for the automated flight control system.

The structure presented can be studied in further detail in [16], [1].

The pitch attitude system block diagram is presented in Fig. 3.

**Fig. 3.** The pitch angle control system [16]

The aircraft is modeled by the block in Fig.3 with the state outputs, [16], [17]:

\[
\dot{x} = \begin{bmatrix} u, \alpha, \theta, q \end{bmatrix}
\]

where:

- \( u \) - aircraft speed along the body axis
- \( \alpha \) - attack angle
- \( \theta \) - pitch angle
- \( q \) - pitch rate

The following variables are also used:

- \( \delta_e \) - elevator deviation
- \( \delta_T \) - throttle deviation
- \( \hat{\theta}_{ref} \) - reference for the pitch angle
- \( \hat{\theta}_m \) - measured pitch angle
- \( \hat{\theta}_e \) - the error as the difference between measured and actual pitch angle

The measured pitch angle provided by vertical gyroscope is the signal that the autopilot is processing.

As described in [16], the transfer function of the pitch attitude control subsystem in closed loop consists of:

1. The aircraft transfer function, for a throttle deviation \( \delta_T = 0 \),

\[
G_{AC}(s) \bigg|_{\delta_T = 0} = \frac{N_{long}(s)}{\Delta_{long}(s)} \tag{22}
\]

where \( N_{long} \) and \( \Delta_{long} \) for the characteristic equation of the aircraft transfer function based on longitudinal dynamics;
2. Vertical gyroscope for which the time response can be assumed constant:
\[ G_{vg} = S_{vg} = ct \] (23)

3. The pitch angle measurement error:
\[ e_\theta (s) = \hat{\theta}_{ref} - \hat{\theta}_m (s) \] (24)

4. The servo mechanism transfer function with \( \tau_s \) - the time response (first order lag when \( \hat{e}_g \) is supplied to the elevators).
\[ G_{ct} (s) = \frac{K_{ct}}{s + 1/\tau_s} \] (25)

The configuration described above is presented in Fig. 2.

\[ \Delta_{ct} (s) = \Delta_{long} (s) + k_{\delta e} \delta e \] (26)

where:
\[ k_{\delta e} = S_{vg} K_{ct} \] (27)

and
\[ N_\theta \delta_e (s) \] - the transfer function for the pitch angle with regards to the elevator control channel.

Therefore (26) can be re-written as:
\[ 1 + k_{\delta e} \delta e \Delta_{long} (s)[s + 1/\tau_s] = 0 \] (28)

From which the open loop transfer function can be determined:
\[ G_{ol} (s) = \frac{N_\theta \delta_e (s)}{\Delta_{long} (s)[\tau_s s + 1]} \] (29)

\[ G_{ol} \] - the open loop transfer function of the pitch attitude control hold system
\[ k_{\delta e} = S_{vg} K_{ct} \tau_s \] (30)

The sensitivity of the vertical gyroscope, \( S_{vg} \) and the delay factor of the servo, \( 1/\tau_s \), are both constants.

3. The Proposed Control Structures and Simulation Results

3.1. Proposed Control Structure for Landing Stabilization

The proposed control structure for landing, using the DMC controller is shown in Fig. 5:

Fig. 5. Control system with a partial state feedback and a feedback loop for the \( \omega \) output using the DMC controller. [2]

In order to show the advantages of the proposed structure, a series of simulations are carried out, taking into account different design parameters.

Given the fact that the control purpose is to ensure a stable command of the aircraft control surfaces, the pitch angle is shown if Fig. 6, with step variation of the reference:

Analyzing the response if can be determined that the apparition of strong oscillations which can damage the control surfaces is prevented.

In this case, the dynamic matrix \( G \) and the DMC gain \( k \) are:

\[ G = \begin{bmatrix} 0 & 0.0209 & 0.0418 & 0.0627 & 0.0836 \\ 0 & 0 & 0.0209 & 0.0418 & 0.0627 \end{bmatrix}^T \]

\[ k = \begin{bmatrix} 0.0258 & 0.0511 & 0.0766 & 0.102 \end{bmatrix} \] (31)

Fig. 7 shows a less satisfactory response due to increased control and prediction horizons (\( N_p = 6, N_c = 2 \)).
In this case, the dynamic matrix $G$ and the DMC gain $k$ are:

$$G = \begin{bmatrix} 0 & 0.0209 & 0.0418 & 0.0627 & 0.0836 & 0.1045 \\ 0 & 0 & 0.0209 & 0.0418 & 0.0627 & 0.0836 \end{bmatrix}^T$$

$$k = \begin{bmatrix} 0.0204 & 0.0405 & 0.0605 & 0.0806 & 0.1007 \end{bmatrix}$$

In comparison, a PI controller was designed for which the parameters were selected in order to compensate the $\lambda_3$ pole. The time response for the PI control is given in Fig. 8, [2]:

$$H(s) = \frac{0.9(s + 0.3973)}{s}$$

Further the influence of the design parameters can be shown. For example, Fig. 9, for $N_p = 5$ and $N_c = 1$, the DMC control structure ensures an accurate control of the aircraft in the specified phase of landing.

$$G = \begin{bmatrix} 0 & 0.0209 & 0.0418 & 0.0623 & 0.0836 \end{bmatrix}^T$$

$$k = \begin{bmatrix} 0.0206 & 0.0413 & 0.0619 & 0.0825 \end{bmatrix}$$

Adjusting the control action weighting factor to $\lambda = 0.7$ and maintaining the same prediction and control horizons as above, we obtain the response from Fig. 10:

$$G = \begin{bmatrix} 0 & 0.0209 & 0.0418 & 0.0627 \end{bmatrix}^T$$

$$k = \begin{bmatrix} 0.0296 & 0.0592 & 0.0888 \end{bmatrix}$$

Considering an increase in the control and prediction horizons the responses from Fig. 11 and Fig. 12:

$$G = \begin{bmatrix} 0 & 0.0209 & 0.0418 & 0.0627 \end{bmatrix}^T$$

$$k = \begin{bmatrix} 0.0296 & 0.0592 & 0.0888 \end{bmatrix}$$

The dynamic matrix $G$ and the DMC gain $k$ are:
Considering the descending speed reference from (13), the following simulations were performed, as in Table 2, [2]:

<table>
<thead>
<tr>
<th>Prediction Horizon</th>
<th>Control Horizon</th>
<th>( \lambda ) - control action weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( N_p = 5 )</td>
<td>( N_c = 1 ) 1</td>
</tr>
<tr>
<td>2</td>
<td>( N_p = 4 )</td>
<td>( N_c = 1 ) 1</td>
</tr>
<tr>
<td>3</td>
<td>( N_p = 4 )</td>
<td>( N_c = 1 ) 0.8</td>
</tr>
<tr>
<td>4</td>
<td>( N_p = 6 )</td>
<td>( N_c = 2 ) 1</td>
</tr>
<tr>
<td>5</td>
<td>( N_p = 6 )</td>
<td>( N_c = 1 ) 1</td>
</tr>
</tbody>
</table>

For the descending speed reference tracking we first present the satisfactory situations which can be seen in Fig. 13 \( (N_p = 5, N_c = 1) \), Fig. 14 \( (N_p = 4, N_c = 1 \) and set-point prediction) and Fig. 15 \( (N_p = 4, N_c = 1, \lambda = 0.8 ) \).

The less satisfactory control situations are presented in Fig. 16 \( (N_p = 6, N_c = 2) \) and Fig. 17 \( (N_p = 6, N_c = 1 \) and set-point prediction).

### 3.2. Proposed Control Problem for Attitude Stabilization

In the following, the use-case for attitude stabilization is presented for the pitch angle control system. The corresponding proposed control structure is discussed in this section. The final control purpose is to ensure a more effective control surface control in aircraft attitude control.

With this in mind, we use the transfer functions of the aircraft N250-100 – prototype 2 (model based on longitudinal dynamics) – considering an altitude \( h = 15000 \) ft and speed \( V_c = 250 \) knots.
The detailed model can be studied in [16]:

\[
N_{\delta e}(s)=S_{\delta e}\left(\frac{s}{376.823}+1\right)\left(\frac{s}{4.2634}+1\right)
\]

\[
N_{\delta e}(s)=S_{\delta e}\left(\frac{s}{91.1168}+1\right)\left(\frac{s}{0.009+0.0381}+1\right)
\]

\[
N_{\theta e}(s)=S_{\theta e}\left(\frac{s}{1.2860}+1\right)\left(\frac{s}{0.0222}+1\right)
\]

The transfer functions represent the elevator channel dynamics. In this particular situation, only the pitch angle control is studied (\(N_{\theta e}(s)\)).

The model uses the following static sensitivity coefficients:

\[
S_{e}=3598.817
\]

\[
S_{\theta e}=-1.992
\]

\[
S_{\delta e}=-8.1443
\]

With the characteristic equation:

\[
\Delta_{\text{long}}(s)=s^4+3.3115s^3+8.1448s^2+1.1604s+0.554
\]

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\Delta_{\text{long}}(s)=s^4+3.3115s^3+8.1448s^2+1.1604s+0.554
\]

With the characteristic equation:

\[
\Delta_{\text{long}}(s)=s^4+3.3115s^3+8.1448s^2+1.1604s+0.554
\]

With the following eigenvalues:

\[
p_{1,2}=-1.6472\pm j2.317
\]

\[
p_{3,4}=-0.0085\pm j0.0824
\]

The N250-100 aircraft gyro is considered \(S_{vg}=1\) (due to fast time response).

The closed loop characteristic equation for the theta angle is:

\[
1+k_{\theta e}G_{cl}(s)=0
\]

Where

\[
k_{\theta e}=S_{vg}K_{cl}\tau_{\theta e}S_{\theta e}=-0.81443K_{cl}
\]

\[
G_{cl}(s)=\left(\frac{s}{1.286}+1\right)\left(\frac{s}{0.0222}+1\right)
\]

\[
G_{cl}(s)=\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\left(\frac{s}{p_3}+1\right)\left(\frac{s}{p_4}+1\right)\tau_{\theta e}+1
\]

\[
G_{cl}(s) - \text{open loop transfer function of the pitch attitude hold system.}
\]

Considering the described model, the open loop and closed loop transfer functions for the pitch attitude control system are given by:

\[
G_{cl}(s)=\frac{158s^2+206.7s+4.512}{s^5+13.31s^4+41.28s^3+81.61s^2+1.659s+0.554}
\]

\[
G_{cl}(s)=\frac{1418s^2+1854s+4.047}{s^5+133.9s^4+412.6s^3+2234s^2+187.1s+4.601}
\]

\[
\theta_{\text{ref}}=\omega_{\text{ref}}t
\]

The proposed control structure based on DMC for the pitch attitude control is presented in Fig. 18:

![Fig. 18. The proposed DMC control structure of the pitch attitude control](image)

The following tests were performed for a step variation of the reference, shown in Table 3:

<table>
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</tr>
<tr>
<td>4</td>
<td>(N_p=5)</td>
<td>(N_c=2)</td>
</tr>
<tr>
<td>5</td>
<td>(N_p=6)</td>
<td>(N_c=1)</td>
</tr>
</tbody>
</table>

First, the tests performed are related to the prediction horizon \(N_p=4\), \(N_c=1\) and a control action weight factor \(\lambda=1\) (shown in Fig. 19), respectively \(N_c=2\) and \(\lambda=0.5\) (shown in Fig. 20):

![Fig. 19. The response of the pitch attitude control system for a step variation of the reference with \(N_c=1\) and \(N_p=4\)](image)

![Fig. 20. Pitch attitude control system response for \(N_c=2\), \(N_p=4\) and \(\lambda=0.5\)](image)
Increasing the prediction horizon to \( N_p = 5 \) and with the variation of the control horizon and \( \lambda \), we obtain the following results presented in Fig. 21 (for \( N_c = 1, \lambda = 1 \)) and Fig. 22 (for \( N_c = 2, \lambda = 0.5 \)). A similar principle is applied for \( N_p = 6 \), for which the results are presented in Fig. 23 (for \( N_c = 1, \lambda = 1 \)).

For a given generic reference trajectory, the pitch attitude hold system behaves as shown in Fig. 25:

![Fig. 25. The response of the pitch attitude control system for a given reference with \( N_c = 1, N_p = 5 \) and \( \lambda = 1 \)](image)

For the pitch attitude hold system the effects of DMC with set-point prediction can be seen in Fig. 26:

![Fig. 26. The pitch attitude control system with set-point prediction for \( N_c = 2, N_p = 5 \) and \( \lambda = 1 \)](image)

The weighting factor of the control action can be modified as can be seen in Fig. 27:

![Fig. 27. The pitch attitude control system with set-point prediction for \( N_c = 2, N_p = 5 \) and \( \lambda = 0.8 \)](image)
4. Conclusions

The case study and simulations show that the DMC algorithm can be used for dedicated single controllers for aircraft attitude control which can be embedded in the automated flight control system. The performance of the proposed structure is superior to that of the classical control approach. Similar considerations are applicable also for the landing DMC structure.

In the attitude stabilization use-case, this paper presents only the calculations for the pitch attitude system. Based on them, the methods for the heading hold and the bank angle hold can be easily determined with the purpose of maintaining a desired reference angle and stabilizing the transition of the aircraft control surfaces.

Choosing the prediction horizon in MPC problems is always a key aspect. The most effective results for both type of structures are obtained for prediction horizon of 5 steps, the rest of the simulations covering truncated less optimal responses and also the influence of the control action weighting factors. For the 5 step prediction horizon, the most satisfactory results are obtained considering the fact that the control purpose is to ensure a protection from oscillations of the control surface movement.

In the last part of the paper, a generic reference trajectory for a longer time interval has been chosen to analyze the effects of the the proposed structure for attitude control. The results have been satisfactory and they have been enhanced when a set-point prediction is included in the algorithm.

5. References